Theory of one-dimensional double-barrier quantum pump in two-frequency signal regime.

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A one-dimensional system with two δ-like barriers or wells bi-chromaticaly oscillating at frequencies ω and 2ω is considered. The alternating signal leads to the direct current across the structure (even in a symmetric system). The properties of this quantum pump are studied in a wide range of the system parameters.

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The quantum pump is a device that generates the stationary current under action of alternating voltage; it is a subject of numerous recent publications (for example,[1,2,3,4,5,6,7,8,9,10]. The quantum pump is essentially analogous to various versions of the photovoltaic effect studied in details from the beginning of the eighties[11,12,13,14,15]. The difference is that the photovoltaic effect is related to the emergence of a direct current in a homogeneous macroscopic medium (the only exception is the mesoscopic photovoltaic effect), while the pump is a microscopic object. From the phenomenological point of view, the emergence of a direct current in the pump is not surprising, since any asymmetric microcontact can rectify ac voltage. However, analysis of adiabatic transport in the quantum-mechanical object leads to new phenomena, such as quantization of charge transport[3]. Just this, analytically solvable, adiabatic approach was utilized in the most of studies of quantum pumps[4,5,6]. In the recent papers we have carried out the extensive study of the simplest model of the one-dimensional quantum pump, containing two delta-like harmonically oscillating barriers/wells. This model demonstrates rich behavior which is ruled by a variety of system parameters. The present paper deals with similar system to which alternating bi-chromatic voltages are applied. The system can be exemplified by a quantum wire with two narrow gates. The stationary bias between the source and the drain is supposedly absent.

The system has a variety of regimes of the pump operation, depending on the system parameters, e.g. frequency and amplitudes. The effect is sensitive to the phase coherence of alternating signals and can exist even in symmetric systems. The stationary current is possible also in the case of different amplitudes of alternating fields. We have studied the system both analytically and numerically. The analytical approach is based on the perturbational (with respect to amplitudes (u_{ij}) of a.c. signal) consideration. The current contains independent contributions caused by u_{ij} and an interference term. The elastic, absorption and emission channels participate in the process. The case of strong alternating signal was studied numerically.

We mostly concentrate on the case of symmetric system as more interesting by its phase sensitivity.

Basic Equations

The considered model is described by the one-dimensional time-dependent potential:

$$\begin{aligned} U(x) = (u_{11}\sin\omega t + u_{12}\sin 2\omega t)\delta(x+d) + \\ (u_{21}\sin\omega t + u_{22}\sin 2\omega t)\delta(x-d). \end{aligned} \tag{1}$$

where t is the time, $2d$ is the distance between δ-barriers (wells); quantities u_{ij} are measured in units of $\hbar/md$ (m is the electron mass); p, E, and ω are the momentum, energy, and frequency measured in units of $\hbar/d$, $\hbar^2/2md^2$, and $\hbar/2md^2$, respectively.

The solution to the Schrödinger equation with the potential (1) is searched in the form

$$\psi = \sum_n \exp\left[-i(E+n\omega)t\right] \begin{cases} \delta_{n,0}\exp\left(\frac{ip_n x}{d}\right) + r_n \exp\left(-\frac{ip_n x}{d}\right), & x < -d, \\ a_n \exp\left(\frac{ip_n x}{d}\right) + b_n \exp\left(-\frac{ip_n x}{d}\right), & -d < x < d, \\ t_n \exp\left(\frac{ip_n x}{d}\right), & x > d. \end{cases} \tag{2}$$

Here, $p_n = \sqrt{p^2 + n\omega}$ and $p = \sqrt{E}$. The wave function (2) corresponds to the wave incident on the barrier from the left. (In the final formulas, we mark the directions of incident waves by the indices "→" and "←"). The form of solution (2) corresponds to absorption (for $n > 0$) or emission ($n < 0$) of n field quanta by an electron after interaction with the vibrating barriers; $n = 0$ relates to the elastic process. Quantities t_n and r_n give the corresponding amplitudes of transmission (reflection). If the value of p_n becomes imaginary, the waves moving away from the barriers should be treated as damped waves, so that $\mathrm{Im} p_n > 0$.

The transmission amplitudes obey the equations: $t_n =$

$$
\begin{pmatrix}
u_{12}u_{22}g_{n-2} \\
u_{12}u_{21}g_{n-2}+u_{11}u_{22}g_{n-1} \\
-i\,u_{12}e^{-2ip_{n-2}}+u_{11}u_{21}g_{n-1}-i\,u_{22}e^{-2ip_n} \\
-u_{12}u_{21}g_{n-2}-i\,u_{11}e^{-2ip_{n-1}}-i\,u_{21}e^{-2ip_n}-u_{11}u_{22}g_{n+1} \\
-u_{12}u_{22}\left(g_{n-2}+g_{n+2}\right)-u_{11}u_{21}\left(g_{n-1}+g_{n+1}\right)+2ip_n e^{-2ip_n} \\
-u_{12}u_{21}g_{n+2}+i\,u_{11}e^{-2ip_{n+1}}+i\,u_{21}e^{-2ip_n}-u_{11}u_{22}g_{n-1} \\
i\,u_{12}e^{-2ip_{n+2}}+u_{11}u_{21}g_{n+1}+i\,u_{22}e^{-2ip_n} \\
u_{12}u_{21}g_{n+2}+u_{11}u_{22}g_{n+1} \\
u_{12}u_{22}g_{n+2}
\end{pmatrix}
\bullet
\begin{pmatrix}
T^{\rightarrow}_{n-4} \\ T^{\rightarrow}_{n-3} \\ T^{\rightarrow}_{n-2} \\ T^{\rightarrow}_{n-1} \\ T^{\rightarrow}_{n} \\ T^{\rightarrow}_{n+1} \\ T^{\rightarrow}_{n+2} \\ T^{\rightarrow}_{n+3} \\ T^{\rightarrow}_{n+4}
\end{pmatrix}
=2ipe^{-ip}\delta_{n,0}, \qquad (3)
$$

and

$$
\begin{pmatrix}
u_{12}u_{22}g_{n-2} \\
u_{11}u_{22}g_{n-2}+u_{12}u_{21}g_{n-1} \\
-i\,u_{22}e^{-2ip_{n-2}}+u_{11}u_{21}g_{n-1}-i\,u_{12}e^{-2ip_n} \\
-u_{11}u_{22}g_{n-2}-i\,u_{21}e^{-2ip_{n-1}}-i\,u_{11}e^{-2ip_n}-u_{12}u_{21}g_{n+1} \\
-u_{12}u_{22}\left(g_{n-2}+g_{n+2}\right)-u_{11}u_{21}\left(g_{n-1}+g_{n+1}\right)+2ip_n e^{-2ip_n} \\
-u_{11}u_{22}g_{n+2}+i\,u_{21}e^{-2ip_{n+1}}+i\,u_{11}e^{-2ip_n}-u_{12}u_{21}g_{n-1} \\
i\,u_{22}e^{-2ip_{n+2}}+u_{11}u_{21}g_{n+1}+i\,u_{12}e^{-2ip_n} \\
u_{11}u_{22}g_{n+2}+u_{12}u_{21}g_{n+1} \\
u_{12}u_{22}g_{n+2}
\end{pmatrix}
\bullet
\begin{pmatrix}
T^{\leftarrow}_{n-4} \\ T^{\leftarrow}_{n-3} \\ T^{\leftarrow}_{n-2} \\ T^{\leftarrow}_{n-1} \\ T^{\rightarrow}_{n} \\ T^{\leftarrow}_{n+1} \\ T^{\leftarrow}_{n+2} \\ T^{\leftarrow}_{n+3} \\ T^{\leftarrow}_{n+4}
\end{pmatrix}
=2ipe^{-ip}\delta_{n,0}. \qquad (4)
$$

Here, $g_n = \sin 2p_n/p_n$.

Provided that electrons from the right and left of the pump are in equilibrium, and that they have identical chemical potentials μ, the stationary current is

$$J=\frac{e}{\pi\hbar}\int dE\sum_n(|T^{\rightarrow}_n|^2-|T^{\leftarrow}_n|^2)f(E)\theta(E+n\omega), \quad (5)$$

where $f(E)$ is the Fermi distribution function, and $\theta(x)$ is the Heaviside step function. The current is determined by the transmission coefficients with real p_n only.

At a low temperature, it is convenient to differentiate the current with respect to the chemical potential:

$$\mathcal{G}=e\frac{\partial}{\partial\mu}J=G_0\sum_n\theta(\mu+n\omega)(|T^{\rightarrow}_n|^2-|T^{\leftarrow}_n|^2)_{p=p_F}. \quad (6)$$

Here $G_0 = e^2/\pi\hbar$ is the conductance quantum and p_F is the Fermi momentum. The resultant quantity $\mathcal{G}$ can be treated as a two-terminal photoconductance (the conductance for simultaneous change of chemical potentials of source and drain).

Perturbations theory

In low-amplitude limit the stationary current (its derivative $\mathcal{G}$) is proportional to

$$\mathcal{G}\propto\alpha_1u_{11}u_{21}+\alpha_2u_{12}u_{22}+\alpha_3u_{11}^2u_{22}+\alpha_4u_{21}^2u_{12}.$$

Let's consider the case of symmetric system with $u_{11} = -u_{21} = u$, $u_{12} = -u_{22} = v$. The systems symmetry leads to the dependence $\mathcal{G} \propto u^2v$. This contribution arises from corresponding terms in the transmission coefficients $T_0, T_{\pm1}, T_{\pm2}$:

$$
\begin{aligned}
T_0 &\propto 1 + A_0u^2v, \\
T_{\pm1} &\propto A_{\pm1}u + B_{\pm1}uv, \\
T_{\pm2} &\propto A_{\pm2}u^2 + B_{\pm2}v.
\end{aligned} \qquad (7)
$$

The quantities A_0, $A_{\pm1}$, $B_{\pm1}$, $A_{\pm2}$, $B_{\pm2}$ depend on $g_{\pm1}$, $g_{\pm2}$ which contains one- and two-photon singularities. The calculations support this dependence.

Numerical results

The figures 1-5 show typical plots of the derivative of the stationary current with respect to the Fermi energy $\partial J/\partial E_F = \mathcal{G} \times 2e^2/h$ as a function of the Fermi momentum (Figs. 1-3), the amplitude (Fig. 4) and the frequency (Fig. 5).

The figures 1-3 present the quantity $\mathcal{G}$ as a function of the Fermi momentum for $u_{21} = u_{22} = -1$, $\omega = 2$ and $u_{11} = u_{12} = 1$ (Fig. 1), 2 (Fig. 2). The peaks of the curves correspond to the multi-photon threshold resonances with zero energy state. The increasing of $u_{11} = u_{12}$ leads to the strengthening of multi-photon singularities.

The curve in the Fig. 1 corresponds to antipodal signals with same amplitudes ($u_{12} = -u_{22}$), that demonstrate the importance of the phase coherency of the signals (in such conditions the stationary current in the same system under monochromatic voltage is vanishing[9]). The current oscillates with the Fermi momentum due to the interference of the electron waves in the structure. Besides, the current possesses singularities caused by the

resonances with the zero energy state and their photon repetitions.

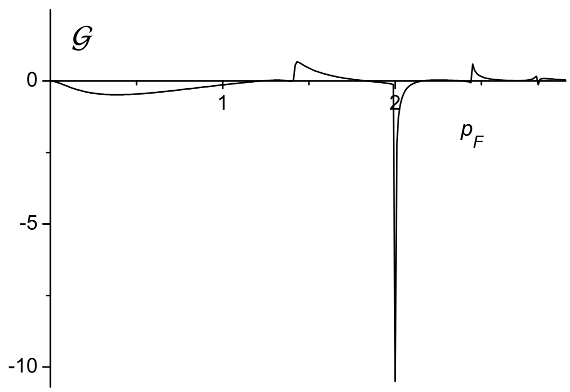

FIG. 1: The derivative of the $\mathcal{G}$ *versus* the Fermi momentum. Here $u_{11} = u_{12} = 1,\ u_{21} = u_{22} = -1,\ \omega = 2$. The left delta-function oscillates from the barrier to zero hight, the right delta-function corresponds to well whose depth changes from zero value

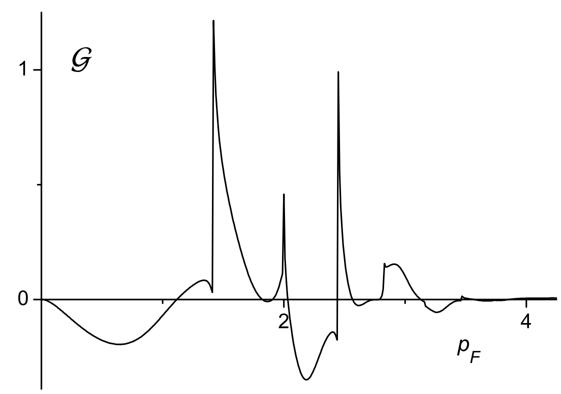

FIG. 2: The dependence of $\mathcal{G}$ on the Fermi momentum. We set $u_{11} = u_{12} = 2,\ u_{21} = u_{22} = -1,\ \omega = 2$.

The figure 3 shows the behavior of $\mathcal{G}$ for large enough frequency where the only threshold singularity exists in the concerned range of Fermi momentum values.

The figure 4 depicts $\mathcal{G}$ *versus* u_{11} for the symmetric system ($u_{11} = u_{12} = -u_{21} = -u_{22}$). These values are chosen so that there should be no current caused by harmonic signals only (if $u_{11} = -u_{21} \neq 0, u_{12} = u_{22} = 0$ or $u_{11} = u_{21} = 0, u_{12} = -u_{22} \neq 0$).

Besides, the current vanishes in the adiabatic approximation which is commonly used for consideration of quantum pumps. In the adiabatic regime the charge transfer per a cycle is proportional to the area covered by two parameters in the phase space. In our case the parameters are $\{u_1(t), u_2(t)\}$. If $u_{11} = u_{12} = -u_{21} = -u_{22}$ the trajectory in the phase space $\{u_1(t), u_2(t)\}$ is a straight line and covers zero area (see insert). Hence in the adiabatic ($\omega \to 0$) approximation $\mathcal{G}/\omega \to 0$.

At low u_{ij} $\mathcal{G} \propto u_\omega^2 u_{2\omega}$. This behavior corresponds to coherent photovoltaic effect[16]. At large u_{ij} the amplitude dependence exhibits distinctive oscillations which

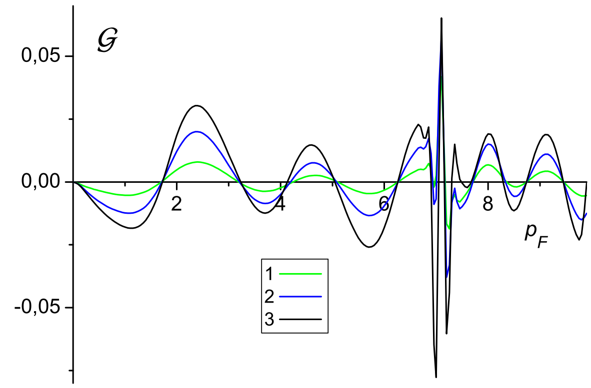

FIG. 3: $\mathcal{G}$ *versus* the Fermi momentum; $u_{11} = u_{12} = 2,\ u_{21} = u_{22} = -1,\ \omega = 50$ and different $u_{11} = u_{12}$. The singularity at $p_F = 7.1$ corresponds to the single-photon threshold $p_F = \sqrt{\omega}$.

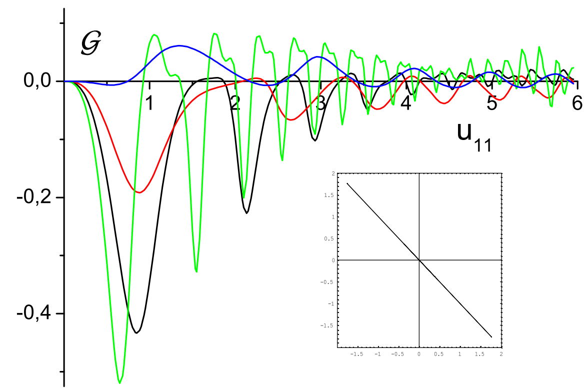

FIG. 4: The dependence of $\mathcal{G}$ on the amplitudes $u_{11} = u_{12} = -u_{21} = -u_{22}$ for $p_F = 0.7$ and different signal frequencies. For $u_{ij} \to 0$, $\mathcal{G} \propto u_{11}^3$. Insert: trajectory in the phase space $\{u_1(t), u_2(t)\}$. The enclosed area in the phase space is zero.

are determined by the commensurability of the characteristic wavelength of excited electrons with the distance between delta-functions. The curves for different frequencies demonstrate splitting and beating of oscillations. Their amplitude decays with u_{ij}. The oscillations period changes with the frequency.

The figure 5 demonstrates the dependence of $\mathcal{G}$ on the frequency. The complicated structure of $\mathcal{G}$ in the low frequency region is explained by multi-photon resonances reducing for larger frequencies.

Conclusions

The alternating voltage produces the stationary current by pumping electrons between leads $x < -d$ and $x > d$. The effect is sensitive to the phase coherence of alternating signals and can exist even in symmetric systems. The stationary current is possible also in the case of different amplitudes of alternating fields.

We have studied the system both analytically and numerically. The analytical approach is based on the perturbational (with respect to amplitudes of a.c. signal)

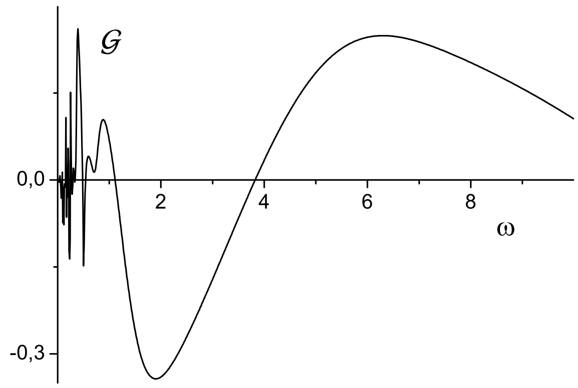

FIG. 5: The dependence of $\mathcal{G}$ on the frequency for $u_{11} = u_{12} = 1$, $u_{21} = u_{22} = -1$, $p_F = 0.7$.

consideration. The current contains independent contributions caused by u_{ij} and an interference term. The elastic, absorption and emission channels participate in the process. The case of strong alternating signal was studied numerically.

Our calculations show that the stationary current is a sophisticated function of the parameters, that reflects the interference effects, presence of virtual states, and threshold singularities.

The motivation to consider the symmetric case is that in this case the stationary current caused by a single frequency is suppressed. Only two intercoherent frequencies can cause the stationary current. Hence, the system with specially symmetrized barriers can be used for heterodyning of signals with ω and 2ω frequencies and for frequency binding of similar laser sources. The controllability of the system permits to realize the symmetric conditions intentionally with good accuracy.

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[1] M. Switkes, C.M. Marcus, K. Campman, and A.C. Gossard, Science **283** 1905 (1999).
[2] S. W. Kim, Int.J.Mod.Phys. B **18**, 3071 (2004).
[3] D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
[4] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
[5] M. Moskalets and M. Büttiker, Phys. Rev. B **66**, 205320 (2002).
[6] M. Moskalets and M. Büttiker, Phys. Rev. B **69** 205316 (2004).
[7] Shi-Liang Zhu and Z. D. Wang, Phys. Rev. B **65**, 155313 (2002).
[8] S. W. Kim, Phys. Rev. B **66**, 235304 (2002).
[9] L. S. Braginskii, M. M. Makhmudian, and M. V. Entin, Zh. Eksp. Teor. Fiz. **127**, 1046 (2005)[JETP **100**, 920 (2005)].
[10] M. M. Mahmoodian, L. S. Braginsky, M. V. Entin, Phys. Rev. B **74**, 125317(2006).
[11] V.I. Belinicher and B.I. Sturman, Usp. Fiz. Nauk **130**, 415 (1980)[Sov. Phys. Usp. **23**, 199 (1980)].
[12] M. D. Blokh, L. I. Magarill, and M. V. Entin, Fiz. Tekh. Poluprovodn. (Leningrad) **12**, 249 (1978)[Sov. Phys. Semicond. **12**, 143 (1978)].
[13] E. M. Baskin, L. I. Magarill, and M. V. Entin, Fiz. Tverd. Tela (Leningrad) **20**, 2432 (1978)[Sov. Phys. Solid State **20**, 1403 (1978)].
[14] E. L. Ivchenko and G. E. Pikus, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 640 (1978)[JETP Lett. **27**, 604 (1978)].
[15] More recent study of photovoltaic effect in artificial 2D lattices of antidots, see for example: A. D. Chepelianskii, and D. L. Shepelyansky, Phys. Rev. B **71**, 052508 (2005); G. Cristadoro, and D. L. Shepelyansky, Phys. Rev. E **71**, 036111 (2005); A. D. Chepelianskii, cond-mat/0509679; M.V. Entin, L.I. Magarill, Phys. Rev. B **73**, 205206 (2006).
[16] E. M. Baskin, M. V. Entin, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 554 (1988)[JETP 48, 601 (1988)]; M.V. Entin, Fiz. Tekh. Poluprovodn. (Leningrad) 23, 1066 (1989)[Sov. Phys. Semicond. 23, 664 (1989)].